
UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
Sidang Akademik 2003/2004

April 2004

JIM 414 – Pentaabiran Statistik

Masa : 3 jam

Sila pastikan bahawa kertas peperiksaan ini mengandungi **DUA PULUH DUA** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.

Jawab SEMUA soalan.

Baca arahan dengan teliti sebelum anda menjawab soalan.

Setiap soalan diperuntukkan 100 markah.

...2/-

1. (a) X_1, X_2, \dots, X_n adalah suatu sampel rawak daripada taburan Bernoulli (θ).

Takrifkan $\bar{X}_n = \frac{1}{n} \sum_{i=1}^n X_i$. Buktikan

- (i) $\bar{X}_n \xrightarrow{p} p$
 (ii) $1 - \bar{X}_n \xrightarrow{p} 1 - p$

(50 markah)

- (b) (i) $X \sim \chi_p^2$. Tunjukkan X juga tertabur secara $F_{1,p}$.
 (ii) $X^2 \sim F_{1,p}$. Tunjukkan X tertabur secara t_p .

(20 markah)

- (c) Daripada sampel rawak bersaiz 20 yang diambil daripada taburan normal kita perolehi

$$\sum_{i=1}^{20} x_i = 257.6 \text{ dan } \sum_{i=1}^{20} x_i^2 = 5234.17.$$

Dapatkan selang keyakinan 90% bagi σ^2 .

(30 markah)

2. (a) X_1, \dots, X_n adalah sampel rawak daripada taburan Poisson (θ)

$$f(x, \theta) = \frac{e^{-\theta} \theta^x}{x!} I_{\{0,1,2,\dots\}}(x), \theta > 0. \text{ Katakan } \bar{X}_n = \frac{1}{n} \sum_{i=1}^n X_i.$$

- (i) Tentukan sama ada $S_n^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X}_n)^2$ penganggar saksama ataupun pincang bagi θ .
 (ii) Tentukan hal yang sama bagi \bar{X}_n .
 (iii) Selain daripada memerhatikan kesaksamaan, bagaimanakah kita boleh menilai S_n^2 dan \bar{X}_n sebagai penganggar θ ? Huraikan cara penilaian ini.

(50 markah)

...3/-

- (b) X_1, \dots, X_n adalah sampel rawak daripada taburan Poisson (θ). Adakah

$$\sum_{i=1}^n X_i \text{ lengkap? Huraikan.}$$

(20 markah)

- (c) X_1, \dots, X_n adalah sampel rawak daripada taburan $N(\mu, \sigma^2)$. Pertimbangkan

$$\text{statistik } Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}.$$

- (i) Sahkan yang statistik ini ialah kuantiti pangsaan.
- (ii) Apakah yang boleh dibina dengan kuantiti pangsaan ini? Huraikan

(30 markah)

3. (a) X_1, \dots, X_n adalah sampel rawak daripada taburan gamma (4, θ). Takrifkan

$$\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i \text{ Terbitkan}$$

- (i) selang keyakinan hampiran 95% bagi θ apabila $n = 100$.
- (ii) Selang keyakinan 95% bagi θ apabila $n = 5$.

(50 markah)

- (b) $H_0 : p = 0.1$ lawan $H_1 : p \neq 0.1$.

X_1, X_2, \dots, X_{10} adalah sampel rawak daripada taburan Bernoulli (p).

Sekiranya $Y = \sum_{i=1}^{10} X_i$ digunakan untuk menentukan rantau genting berbentuk

$$Y \geq 6,$$

- (i) nilaikan kebarangkalian ralat Jenis I.
- (ii) nilaikan kebarangkalian-kebarangkalian ralat Jenis II jika alternatif-alternatif kepada H_0 sebenarnya adalah $p = 0.2$ dan $p = 0.4$.

(50 markah)

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4. (a) X_1, \dots, X_n adalah sampel rawak daripada taburan yang berfungsi ketumpatan

$$f(x; \theta) = (2\theta x + 1 - \theta)I_{(0,1)}(x), -1 \leq \theta \leq 1.$$

Katakan kita ingin menguji $H_0 : \theta \leq \theta_0$ dan $H_1 : \theta > \theta_0$.

- (i) Dapatkan rantau genting paling berkuasa secara seragam saiz- α jika ia wujud.
- (ii) Cari statistik $T = u(X_1, \dots, X_n)$ yang mana nisbah $L(\theta_1; x_1, \dots, x_n)/L(\theta_2; x_1, \dots, x_n)$ merupakan fungsi berekanada bagi $t = u(x_1, \dots, x_n)$, $\theta_1 \in \Theta_0$ dan $\theta_2 \in \Theta_1$, jika ianya wujud.
- (iii) Bolehkah kita menggunakan $f(x; \theta) = a(\theta) b(x) \exp [c(\theta)d(x)]$ untuk mendapatkan rantau genting paling berkuasa secara seragam saiz- α ? Jelaskan.
- (iv) Dapatkan rantau genting saiz- α ujian nisbah kebolehdian hipotesis ini. Adakah (i) sama dengan (iv)?

(80 markah)

- (b) Sekeping syiling dilempar 500 kali. Dua ratus lapan puluh kepala dan 220 bunga muncul. Adakah syiling ini adil? Jelaskan.

(20 markah)

...5/-

5. (a) X_1, \dots, X_n adalah sampel rawak daripada taburan seragam (a, b) . Dapatkan taburan statistik tertib ke- j sampel ini.

(25 markah)

- (b) X_1, \dots, X_n adalah sampel rawak daripada taburan Gamma $\left(\frac{1}{2}, \frac{\sigma^2}{2}\right)$.

Dapatkan batas bawah Cramer-Rao bagi σ^2 .

(25 markah)

- (c) X_1, X_2 adalah sampel rawak daripada taburan seragam $(0, \theta + 1)$. Untuk menguji $H_0: \theta = 0$ lawan $H_1: \theta > 0$, dua ujian dipertimbangkan:

I: Tolak H_0 jika $X_1 > 0.95$

II: Tolak H_0 jika $X_1 + X_2 > C$.

Cari C supaya saiz Ujian II sama dengan saiz Ujian I.

(25 markah)

- (d) $L(x)$ dan $U(x)$ memenuhi $P(L(X) \leq \theta) = \alpha_1$ dan $P(U(X) \geq \theta) = \alpha_2$.
Nilaikan $P(L(X) \leq \theta \leq U(X))$.

(25 markah)

Lampiran

1. $\lim_{n \rightarrow \infty} F_n(z) = \Phi(z)$
2. $E[cX] = c E[X]$
3. $\text{Var}(aX + b) = a^2 \text{Var}(X)$
4. $E[\bar{X}] = \mu$
5. $\text{Var}(\bar{X}) = \frac{\sigma^2}{n}$
6. $P(|X - \mu| \geq \varepsilon) \leq \frac{\sigma^2}{\varepsilon^2}$
7. $Z_n = \frac{\sqrt{n}(\bar{X}_n - \mu)}{\sigma}$
8. $M_{\sum X_i}(t) = [M_X(t)]^n$
9. $M_{\bar{X}}(t) = [M(t/n)]^n$
10. $g_1(y) = n[1 - F(y)]^{n-1} f(y)$
11. $g_\alpha(y) = \frac{n!}{(\alpha-1)!(n-\alpha)!} [F(y)]^{\alpha-1} f(y) [1 - F(y)]^{n-\alpha}$

12.
$$g_{\alpha, \beta}(x, y) = \frac{n!}{(\alpha-1)!(\beta-\alpha-1)!(n-\beta)!} [F(x)]^{\alpha-1} f(x) [F(y) - F(x)]^{\beta-\alpha-1} f(y) [1 - F(y)]^{n-\beta},$$

 $\alpha < \beta$
13.
$$g_n(y) = n[F(y)]^{n-1} f(y)$$
14.
$$f_Y(t) = f_X[g^{-1}(t)] |J|$$
15.
$$J = \frac{dg^{-1}(t)}{dt}$$
16.
$$L(\theta; x_1, x_2, \dots, x_n) = \prod_{i=1}^n f(x_i, \theta)$$
17.
$$f(x; \theta) = a(\theta) b(x) \exp [c(\theta) d(x)]$$
18.
$$\text{Var}(T) \geq \frac{[\tau'(\theta)]^2}{nE\left[\left\{\frac{\partial}{\partial \theta} \log f(x; \theta)\right\}^2\right]}$$
19.
$$E\left[\left\{\frac{\partial}{\partial \theta} \log f(x; \theta)\right\}^2\right] = -E\left[\frac{\partial^2}{\partial \theta^2} \log f(x; \theta)\right]$$

Rumus-Rumus

Modul 1

Pelajaran 1

1. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
2. $P(A) = P(A \cap \bar{B}) + P(A \cap B)$
3. $P(\bar{A}) = 1 - P(A)$
4. $n_{Pr} = \frac{n!}{(n-r)!}$
5. $\binom{n}{r} = \frac{n!}{r!(n-r)!}$
6. $N = \frac{n!}{n_1! n_2! \dots n_k!}$

Pelajaran 2

1. $P(A|B) = \frac{P(A \cap B)}{P(B)}$
2. $P(A \cap B) = P(A)P(B)$
3. $P(A) = P(A|B)P(B) + P(A|\bar{B})P(\bar{B})$
4. $P(B_i|A) = \frac{P(A \cap B_i)}{\sum_{j=1}^n P(A \cap B_j)P(B_j)}$

Pelajaran 3

1. $P(a \leq X \leq b) = \int_a^b f(x) dx$
2. $P(a < X < b) = \sum_{a < x < b} p(x)$
3. $F(t) = P(X \leq t)$
4. $P(a < X \leq b) = F(b) - F(a)$

5. $\frac{d}{dt} F(t) = f(t)$
6. $F_Y(t) = F_X(g^{-1}(t))$
7. $F_Y(t) = 1 - F_X(g^{-1}(t))$
8. $f_Y(t) = f_X(g^{-1}(t)) |J|$
9. $J = \frac{dg^{-1}(t)}{dt}$
10. $f_Y(t) = \sum_{i=1}^k f_X(g_i^{-1}(t)) |J_i|$
11. $J_i = \frac{d}{dt} g_i^{-1}(t)$
12. $P_Y(y) = \sum_{x \in A} P_X(x)$

Modul 2

Pelajaran 1

1. $E(X) = \sum_{x \in \text{Julat } X} xp(x)$
2. $1 + x + x^2 + \dots + x^n + \dots = \frac{1}{1-x}, |x| < 1$
3. $1 + 2x + \dots + nx^{n-1} + \dots = \frac{1}{(1-x)^2}, |x| < 1$
4. $E(X) = \int_{-\infty}^{\infty} x f(x) dx$
5. $E(X) = \int_0^{\infty} [1 - f(x)] dx - \int_{-\infty}^0 F(x) dx$
6. $E[G(X)] = \sum_{x \in \text{Julat } X} G(x) p(x)$

7. $E[G(X)] = \int_{-\infty}^{\infty} G(x) f(x) dx$
8. $E[c] = c$
9. $E[cX] = c E[X]$
10. $E[X + c] = E[X] + c$
11. $\text{Var}(X) = E[X - E[X]]^2$
12. $\text{Var}(X) = E[X^2] - \mu_X^2$
13. $\text{Var}(X) = \sum_{x \in \text{Julat } X} x^2 p(x) - \mu_X^2$
14. $\text{Var}(X) = \int_{-\infty}^{\infty} x^2 f(x) dx - \mu_X^2$
15. $\text{Var}(a) = 0$
16. $\text{Var}(aX + b) = a^2 \text{Var}(X)$
17. $F_X(t_k) = k, 0 < k < 1$

Pelajaran 2

1. $m_k = E[X^k]$
2. $m_k = \sum_{x \in \text{Julat } X} x^k p(x)$
3. $m_k = \int_{-\infty}^{\infty} x^k f(x) dx$
4. $\mu_k = E[(X - \mu_X)^k]$
5. $\gamma_1 = \mu_3 / \sigma_X^3$
6. $\gamma_2 = \frac{\mu_4}{\sigma_X^4} - 3$
7. $\mu_{[k]} = E[X(X-1)(X-2) \dots (X-k+1)]$
8. $m(t) = E[e^{tX}]$

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9. $m(t) = \sum_{x \in \text{Julat } X} e^{tx} p(x)$
10. $m(t) = \int_{-\infty}^{\infty} e^{tx} f(x) dx$
11. $m_Y(t) = E[e^{tg(X)}]$
12. $m_Y(t) = \sum_{x \in \text{Julat } X} e^{tg(x)} p(x)$
13. $m_Y(t) = \int_{-\infty}^{\infty} e^{tg(x)} f(x) dx$
14. $m_Y(t) = e^{bt} m_X(at)$
15. $m^{(i)}(0) = m_i$
16. $k(t) = \ell n m(t)$
17. $\psi(t) = E[t^X]$
18. $f(t) = \sum_{i=0}^{\infty} \frac{f^{(i)}(a)}{i!} (t-a)^i$
19. $\psi^{(i)}(0) = i! p(i)$
20. $P(|X| \geq a) < \frac{1}{a^2} E[X^2]$
21. $P(|X - \mu| \geq a\sigma) \leq \frac{1}{a^2}$
22. $P(|X - \mu| < a\sigma) \geq 1 - \frac{1}{a^2}$
23. $P(X \geq a) \leq \frac{E[X]}{a}$
24. $E[X^n] = \int_0^{\infty} nx^{n-1} (1 - F(x)) dx$

Pelajaran 3

$$1. \quad (i) \quad p(x) = \begin{cases} q, & x = 0 \\ p, & x = 1 \\ 0, & \text{di tempat lain} \end{cases}$$

$X \sim \text{Bernoulli}(p)$

$$(ii) \quad E[X] = p$$

$$(iii) \quad \text{Var}(X) = pq$$

$$(iv) \quad m(t) = q + pe^t$$

$$2. \quad (i) \quad p(x) = \begin{cases} \binom{n}{x} p^x q^{n-x}, & x = 0, 1, 2, \dots, n \\ 0, & \text{di tempat lain} \end{cases}$$

$X \sim \text{Binomial}(n, p)$

$$(ii) \quad E[X] = np$$

$$(iii) \quad \text{Var}(X) = npq$$

$$(iv) \quad m(t) = (q + pe^t)^n$$

$$3. \quad (i) \quad p(x) = \begin{cases} \frac{\binom{K}{x} \binom{N-K}{n-x}}{\binom{N}{n}}, & x = 0, 1, 2, \dots, n \\ 0, & \text{di tempat lain} \end{cases}$$

$X \sim \text{hipergeometri}(N, k, n)$

$$(ii) \quad E[X] = \frac{nK}{N}$$

$$(iii) \quad \text{Var}(X) = \frac{nK(N-K)(N-n)}{N^2(N-1)}$$

$$4. \quad (a + b)^n = \sum_{i=0}^n \binom{n}{i} a^i b^{n-i}$$

$$5. \quad (i) \quad p(x) = \begin{cases} q^{x-1}p, & x = 1, 2, 3, \dots \\ 0, & \text{di tempat lain} \end{cases}$$

$X \sim \text{geometri } (p)$

$$(ii) \quad E[X] = 1/p$$

$$(iii) \quad \text{Var}(X) = q/p^2$$

$$(iv) \quad m(t) = \frac{pe^t}{1 - qe^t}$$

$$6. \quad (i) \quad p(x) = \begin{cases} \binom{x-1}{r-1} p^r q^{x-r}, & x = r, r+1, r+2, \dots \\ 0, & \text{di tempat lain} \end{cases}$$

$X \sim \text{negatif binomial } (r, p)$

$$(ii) \quad E[X] = r/p$$

$$(iii) \quad \text{Var}(X) = rq/p^2$$

$$(iv) \quad m(t) = \left[\frac{pe^t}{1 - qe^t} \right]^r$$

$$7. \quad (i) \quad p(x) = \begin{cases} e^{-\lambda} \frac{\lambda^x}{x!}, & x = 0, 1, 2, \dots \\ 0, & \text{di tempat lain} \end{cases}$$

$X \sim \text{Poisson } (\lambda)$

$$(ii) \quad E[X] = \lambda$$

$$(iii) \quad \text{Var}(X) = \lambda$$

$$(iv) \quad m(t) = e^{\lambda(e^t - 1)}$$

$$8. \quad \lim_{x \rightarrow 0} (1+x)^{1/x} = e$$

$$9. \quad \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$$

$$10. \quad \lim_{x \rightarrow 0} (1+ax)^{1/x} = e^a$$

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Pelajaran 4

$$1. \quad (i) \quad f(x) = \begin{cases} \frac{1}{b-a}, & a < x < b \\ 0, & \text{di tempat lain} \end{cases}$$

$X \sim \text{seragam}(a, b)$

$$(ii) \quad E[X] = \frac{a+b}{2}$$

$$(iii) \quad \text{Var}(X) = \frac{(b-a)^2}{12}$$

$$(iv) \quad m(t) = \frac{e^{bt} - e^{at}}{t(b-a)}$$

$$2. \quad (i) \quad f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}, \quad -\infty < x < \infty$$

$X \sim N(\mu, \sigma^2)$

$$(ii) \quad E[X] = \mu$$

$$(iii) \quad \text{Var}(X) = \sigma^2$$

$$(iv) \quad m(t) = e^{\mu t - \frac{1}{2}\sigma^2 t^2}$$

$$3. \quad \lim_{n \rightarrow \infty} P \left[a \leq \frac{S_n - np}{\sqrt{npq}} \leq b \right] \rightarrow P(Z \geq a) - P(Z > b)$$

$$4. \quad \lim_{\lambda \rightarrow \infty} P \left[a \leq \frac{X - \lambda}{\sqrt{\lambda}} < b \right] \rightarrow P(Z > a) - P(Z \geq b)$$

$$5. \quad (i) \quad f(x) = \begin{cases} \lambda e^{-\lambda}, & x \geq 0 \\ 0, & \text{di tempat lain} \end{cases}$$

$X \sim \text{eksponen}(\lambda)$

$$(ii) \quad E[X] = 1/\lambda$$

$$(iii) \quad \text{Var}(X) = 1/\lambda^2$$

$$(iv) \quad m(t) = \frac{\lambda}{\lambda - t}$$

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$$6. \quad \Gamma(n) = \int_0^{\infty} x^{n-1} e^{-x} dx$$

$$7. \quad \Gamma(n) = (n-1) \Gamma(n-1)$$

$$8. \quad \Gamma(n) = (n-1)!$$

$$9. \quad (i) \quad f(x) = \begin{cases} \frac{\lambda^n x^{n-1}}{\Gamma(n)} e^{-\lambda x}, & x > 0 \\ 0, & \text{di tempat lain} \end{cases}$$

$X \sim \text{Gamma}(n, \lambda)$

$$(ii) \quad E[X] = n/\lambda$$

$$(iii) \quad \text{Var}(X) = n/\lambda^2$$

$$(iv) \quad \dot{m}(t) = \left(\frac{\lambda}{\lambda - t} \right)^n$$

$$10. \quad (i) \quad f(x) = \begin{cases} \frac{1}{2^{v/2} \Gamma\left(\frac{v}{2}\right)} x^{v/2-1} e^{-x/2}, & x > 0 \\ 0, & \text{di tempat lain} \end{cases}$$

$X \sim \chi_v^2$

$$(ii) \quad E[X] = v$$

$$(iii) \quad \text{Var}(X) = 2v$$

$$(iv) \quad m(t) = \left(\frac{1}{1-2t} \right)^{v/2}$$

$$11. \quad B(x, y) = \int_0^1 t^{x-1} (1-t)^{y-1} dt$$

$$12. \quad B(x, y) = \int_0^{\infty} \frac{t^{x-1}}{(1+t)^{x+y}} dt$$

$$13. \quad B(x, y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

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$$14. \quad (i) \quad f(x) = \begin{cases} \frac{1}{B(a,b)} x^{a-1} (1-x)^{b-1}, & 0 < x < 1 \\ 0, & \text{di tempat lain} \end{cases} \quad X \sim \text{Beta}(a, b)$$

$$(ii) \quad F_X(p) = \sum_{x=0}^n \binom{n}{x} p^x (1-p)^{n-x}$$

$$(iii) \quad E[X] = \frac{a}{a+b}$$

$$(iv) \quad \text{Var}(X) = \frac{ab}{(a+b+1)(a+b)^2}$$

Modul 3

Pelajaran 1

$$1. \quad P(X \leq x, Y \leq y) = \sum_{t_1 \leq x} \sum_{t_2 \leq y} p(t_1, t_2)$$

$$2. \quad P(X \leq x, Y \leq y) = \int_{-\infty}^x \int_{-\infty}^y f(t_1, t_2) dt_1 dt_2$$

$$3. \quad F(x, y) = P(X \leq x, Y \leq y)$$

$$4. \quad f(x, y) = \frac{\partial^2 F(x, y)}{\partial x \partial y}$$

Pelajaran 2

$$1. \quad p(x) = \sum_y p(x, y)$$

$$2. \quad p(y) = \sum_x p(x, y)$$

$$3. \quad f(x) = \int_{-\infty}^{\infty} f(x, y) dy$$

$$4. \quad f(y) = \int_{-\infty}^{\infty} f(x, y) dx$$

$$5. \quad F(x) = F(x, \infty)$$

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$$6. \quad F(y) = F(\infty, y)$$

$$7. \quad f(x) = \frac{\partial F(x, \infty)}{\partial x}$$

$$8. \quad f(y) = \frac{\partial F(\infty, y)}{\partial y}$$

$$9. \quad p(x | y) = \frac{p(x, y)}{p(y)}$$

$$10. \quad f(x | y) = \frac{f(x, y)}{f(y)}$$

$$11. \quad p(x, y) = p(x) p(y)$$

$$12. \quad f(x, y) = f(x) f(y)$$

Pelajaran 3

$$1. \quad E[g(X, Y)] = \sum_x \sum_y g(x, y) p(x, y)$$

$$2. \quad E[g(X, Y)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x, y) f(x, y) dx dy$$

$$3. \quad E[g_1(X, Y) + g_2(X, Y)] = E[g_1(X, Y)] + E[g_2(X, Y)]$$

$$4. \quad E[h_1(X) h_2(Y)] = E[h_1(X)] E[h_2(Y)]$$

$$5. \quad (i) \quad \text{Cov}(X, Y) = E[X - \mu_X] (Y - \mu_Y)]$$

$$(ii) \quad \text{Cov}(X, Y) = E[XY] - \mu_X \mu_Y$$

$$6. \quad \text{Cov}(aX, bY) = ab \text{Cov}(X, Y) \quad 9 -$$

$$7. \quad \text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y) + 2 \text{Cov}(X, Y)$$

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8. $\text{Var} \left(\sum_{i=1}^n X_i \right) = \sum_{i=1}^n \text{Var} (X_i) + 2 \sum_{i < j} \text{Cov} (X, Y)$
9. $\rho(X, Y) = \frac{\text{Cov} (X, Y)}{\sigma_X \sigma_Y}$
10. $E[g(X, Y) | Y = y] = \sum_x g(x, y) p(x | y)$
11. $E[g(X, Y) | Y = y] = \int_{-\infty}^{\infty} g(x, y) f(x | y) dx$
12. $E[E[X | Y = y]] = E[X]$
13. $E[E[Y | X = x]] = E[Y]$
14. $E[E[g(X) | Y = y]] = E[g(X)]$
15. $E[E[g(Y) | X = x]] = E[g(Y)]$
16. $\text{Var} (X | Y = y) = E[X^2 | Y = y] - (E[X | Y = y])^2$
17. $m(t_1, t_2) = E[e^{t_1 X_1 + t_2 X_2}]$
18. $m(t_1, t_2, \dots, t_n) = E\left[e^{\sum_{i=1}^n t_i X_i}\right]$
19. $m(t_1) = \lim_{t_2 \rightarrow 0} m(t_1, t_2)$
20. $m(t_1, t_2, \dots, t_n) = m(t_1) m(t_2) \dots m(t_n)$

Pelajaran 4

1. (i) $p(x_1, x_2, \dots, x_k) = \frac{n!}{x_1! x_2! \dots x_k!} p_1^{x_1} p_2^{x_2} \dots p_k^{x_k}$
- (ii) $p(x_i) = \binom{n}{x_i} p_i^{x_i} (1 - p_i)^{n - x_i}$
- (iii) $p(x_i, x_j) = \frac{n!}{x_i! x_j! (n - x_i - x_j)!} p_i^{x_i} p_j^{x_j} (1 - p_i - p_j)^{n - x_i - x_j}$
- (iv) $E[X_i X_j] = n(n - 1) p_i p_j$
- (v) $\text{Cov} (X_i, X_j) = -np_i p_j$

...19/-

$$2. (i) f(x,y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \exp \left\{ -\frac{1}{2(1-\rho^2)} \left[\left(\frac{x-\mu_x}{\sigma_x} \right)^2 - 2\rho \left(\frac{x-\mu_x}{\sigma_x} \right) \left(\frac{y-\mu_y}{\sigma_y} \right) + \left(\frac{y-\mu_y}{\sigma_y} \right)^2 \right] \right\},$$

$$-\infty < x < \infty, -\infty < y < \infty$$

$$(ii) f(x|y) = \frac{1}{\sigma_x\sqrt{2\pi(1-\rho^2)}} \exp \left\{ -\frac{1}{2(1-\rho^2)\sigma_x^2} \left[x - \mu_x - \rho \frac{\sigma_x}{\sigma_y} (y - \mu_y) \right]^2 \right\}$$

$$-\infty < x < \infty$$

$$(iii) m(t_1, t_2) = \exp \left[t_1\mu_x + t_2\mu_y + \frac{1}{2} (t_1^2\sigma_x^2 + 2\rho t_1 t_2 \sigma_x \sigma_y + t_2^2\sigma_y^2) \right]$$

$$(iv) E[XY] = \mu_x\mu_y + \rho \sigma_x\sigma_y$$

$$(v) \text{Cov}(X, Y) = \rho \sigma_x\sigma_y$$

Modul 4

Pelajaran 1

$$1. M_k = \frac{1}{n} \sum_{i=1}^n X_i^k$$

$$2. E[M_k] = m_k$$

$$3. \text{Var}(M_k) = \frac{1}{n} [m_{2k} - m_k^2]$$

$$4. E[\bar{X}] = \mu$$

$$5. \text{Var}(\bar{X}) = \frac{1}{n} \sigma^2$$

$$6. S^2 = \frac{1}{(n-1)} \sum_{i=1}^n (X_i - \bar{X})^2$$

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$$7. \quad E[S^2] = \sigma^2$$

$$8. \quad \text{Var}(S^2) = \frac{1}{n} \left(\mu_4 - \frac{(n-3)}{(n-1)} \sigma^4 \right)$$

$$9. \quad \sum_{i=1}^n (X_i - \mu)^2 = \sum_{i=1}^n (X_i - \bar{X})^2 + n(\bar{X} - \mu)^2$$

$$10. \quad \bar{X} - \mu = \frac{1}{n} \sum_{i=1}^n (X_i - \mu)$$

Pelajaran 2

$$1. \quad p(u, v) = p_{X,Y} (g_1^{-1}(u, v), g_2^{-1}(u, v))$$

$$2. \quad f(u, v) = f_{X,Y} (g_1^{-1}(u, v), g_2^{-1}(u, v)) |J|$$

$$3. \quad J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

$$4. \quad f(u, v) = \sum_{i=1}^m |J_i| f_{X,Y} (g_i^{-1}(u, v), h_i^{-1}(u, v))$$

$$5. \quad J_i = \begin{vmatrix} \frac{\partial g_i^{-1}(u, v)}{\partial u} & \frac{\partial g_i^{-1}(u, v)}{\partial v} \\ \frac{\partial h_i^{-1}(u, v)}{\partial u} & \frac{\partial h_i^{-1}(u, v)}{\partial v} \end{vmatrix}$$

$$6. \quad m_{u,v}(t_1, t_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{t_1 g(x,y) + t_2 h(x,y)} f(x,y) dx dy$$

$$7. \quad m_u(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{t g(x,y)} f(x,y) dx dy$$

$$8. \quad (i) \quad f_{u=X+Y}(u) = \int_{-\infty}^{\infty} f_{X,Y}(x, u-x) dx$$

$$(ii) \quad f_{u=X+Y}(u) = \int_{-\infty}^{\infty} f_{X,Y}(u-y, y) dy$$

$$9. \quad (i) \quad f_{u=X-Y}(u) = \int_{-\infty}^{\infty} f_{X,Y}(x, x-u) dx$$

$$(ii) \quad f_{u=X-Y}(u) = \int_{-\infty}^{\infty} f_{X,Y}(u+y, y) dy$$

$$10. \quad (i) \quad f_{u=XY}(u) = \int_{-\infty}^{\infty} \frac{1}{|x|} f_{X,Y}(x, u/x) dx$$

$$(ii) \quad f_{u=XY}(u) = \int_{-\infty}^{\infty} \frac{1}{|y|} f_{X,Y}(u/y, y) dy$$

$$11. \quad f_{u=XY}(u) = \int_{-\infty}^{\infty} |y| f_{X,Y}(uy, y) dy$$

Pelajaran 3

$$1. \quad (i) \quad f(x) = \frac{\Gamma[(n+1)/2]}{\Gamma(n/2)\sqrt{\pi n}} \left(1 + \frac{x^2}{n}\right)^{-(n+1)/2}, -\infty < x < \infty \quad X \sim t_n$$

$$(ii) \quad T = \frac{Z}{\sqrt{V/n}}$$

$$(iii) \quad E[X] = 0$$

$$(iv) \quad \text{Var}[X] = \frac{n}{n-2}$$

$$2. \quad (i) \quad f(x) = \begin{cases} \frac{\Gamma[(m+n)/2]}{\Gamma(m/2)\Gamma(n/2)} \left(\frac{m}{n}\right)^{m/2} \frac{x^{(m-2)/2}}{[1+(m/n)x]^{(m+n)/2}}, & x > 0 \\ 0 & , \text{ di tempat lain} \end{cases} \quad X \sim F_{m,n}$$

$$(ii) \quad F = \frac{U/m}{V/m}$$

$$(iii) \quad E[X] = \frac{n}{n-2}$$

$$(iv) \quad \text{Var}(X) = \frac{2n^2(m+n-2)}{m(n-2)^2(n-4)}$$

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